## *NOTE ON PROBLEM* 374.

BY PROF. ASAPH HALL.

IF the solution of this problem given on p. 30 of the ANALYST, Vol. IX, be correct, it furnishes an easy method for improving numerical tables. Thus, we have only to interpolate the table into the middle, as the phrase is, and then laying aside the original table we have one of equal intervals but more accurate. Again interpolating into the middle we get a table still more accurate than the second one, and having the same arguments as the original, and so on *ad infinitum*. But the solution seems to me erroneous.

If we have a series of values $v$, $v'$, $v''$, &c., all of which have the common probable error $r$, then the probable error of the sum or difference of any two of these values is $r.\sqrt{2}$. Since in interpolating we multiply the difference, $v'-v$, by a factor, $\frac{t}{10}$, the probable error of the correction $=\Delta$, that we add to $v$, is $\frac{rt.\sqrt{2}}{10}$. Now the probable error of $v+\Delta$ is

$$r.\left(1 + \frac{2t^2}{100}\right)^{\frac{1}{2}}.$$

Hence the interpolated value is less correct than the tabular one, and our method of improving the tables fails.

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## *PLANETARY MASS AND VIS VIVA.*

BY PLINY EARLE CHASE, LL. D.

ALL persistent oscillations in elastic media, whether luminous, electric, thermal, atomic, molecular or cosmical, MUST BE harmonic.

The fundamental harmonies of oscillatory movement in the luminiferous æther, must involve simple functions of the velocity of light.

In applying the oscillatory equation,

$$t = \pi\sqrt{\frac{l}{g}},$$

at the centre of gravity of a stellar system, let $t$ represent the duration of an oscillation or half-rotation, $g$ the acceleration of gravity at the stellar equatorial surface, $\pi^2 l$ the stellar modulus of light or the height of a homogeneous æthereal atmosphere which would propagate undulations with the velocity of light. Then, if the stellar rotary oscillation is due to the reaction of cosmical inertia against ætherial influence, *gt is equivalent to the velocity of light*, $v_\lambda$.

Coulomb's torsional formula may be applied to Sun, by taking Sun's equatorial semidiameter, $r_0$, as the radial unit:—

$$f = \frac{m}{2} = \frac{W}{2} \cdot \frac{\pi^2 a^2 r}{gt^2};\ \pi^2 a^2 r_0 = \pi^2 l = gt^2;\ gt = v_\lambda. \qquad (1)$$

If sun's apparent semidiameter is 961″.83, Earth's semi axis major ($\zeta_3$) $= 214.45 r_0$; $g$ at Sun's equatorial surface ($g_0$) $= .0000003909446 r_0$; $v_\lambda = \zeta_3 \div 497.827 = .4307721 r_0$. Therefore, from (1), $t = 1101875$ sec.; solar rotation $= 2t = 2203750$ sec. $= 25.506$ days.

*Vis viva* may be represented by orbital areas, as well as by distances of projection against uniform resistance. The virtual areas of synchronous planetary reaction, or the mean instantaneous areas which a particle at Sun's surface *tends* to describe about any given planet, vary as $\sqrt{(mr)}$. The doctrine of conservation of energy supplements Laplace's two laws of constancy by a third, viz.:—*The sum of all the instantaneous virtual areas in a system will always remain invariable.*

The laws of the chief centre of condensation, Earth, which is the central and controlling planet in the dense belt, exerts important harmonic influences. If Earth were rotating with the speed which a coincidence of Laplace's limit with its equatorial surface would give, its time of rotation would be $2\pi \sqrt{(r \div g)} = 5073.8$ seconds. Its coefficient of orbital retardation is, therefore, $\varkappa = 86164.1 \div 5073.8 = 16.9822$. In an expanding or condensing nebula, the atmospheric radius varies as the $\frac{4}{3}$ power of the nucleal radius; $\varkappa^{\frac{4}{3}} = 43.651$.

Herschel's locus of incipient subsidence, in the controlling two-planet belt, or Saturn's secular aphelion, is 1.0843289 times the outer limiting locus of the belt (Stockwell, *Smithson. Contrib.*, 232, p. 38); $\varkappa^{\frac{4}{3}} \div 1.0843289 = 40.256$, which is, approximately, the ratio of the instantaneous virtual area at the inner locus of the controlling belt, to the virtual area at the chief centre of condensation. The tendency of exponents, in elastic media, to become coefficients of harmonic *vis viva*, is shown in the following table:—

| Harmonic Areas. | | Mean Virtual Areas. | | Difference. |
|---|---|---|---|---|
| $\alpha = \varkappa^{\frac{4}{3}}$ | 40.256 | Jupiter | 40.587 | —.331 |
| $\beta = \frac{3}{4}\alpha$ | 30.192 | Saturn | 30.063 | +.129 |
| $\gamma = \frac{3}{4}\beta$ | 22.644 | Neptune | 22.675 | —.031 |
| $\varkappa = \frac{3}{4}\gamma$ | 16.983 | Uranus | 16.782 | +.201 |
| $\varepsilon$ | 1.000 | Earth | 1.000 | .000 |
| $\zeta = \frac{3}{4}\varepsilon$ | .750 | Venus | .749 | +.001 |
| $\delta = \frac{3}{4}\zeta$ | .562 | ... | ... | .. |
| $\eta = \frac{3}{4}\delta$ | .422 | Mars | .404 | +.018 |
| $\theta = \frac{3}{4}\eta$ | .316 | ... | ... | .. |
| $\iota = \frac{3}{4}\theta$ | .237 | ... | ... | .. |
| $\lambda = \frac{3}{4}\iota$ | .178 | Mercury | .162 | +.016 |

The percentage of difference between the harmonic and virtual areas is, respectively, $\frac{5}{6}$ of .01, $\frac{3}{7}$ of .01, $\frac{1}{7}$ of .01, $\frac{6}{5}$ of .01, $\frac{1}{7}$ of .01, .045, .099. In testing the combined harmonic influences of a *vera causa*, subject to internal perturbations, there is room for a possible deviation of 50 per cent, and a probable deviation of 25 per cent. The combined probability of the dependence of the above approximations upon æthereal influence is, therefore, $(25 \div \frac{5}{6} = 30) \times \frac{175}{3} \times 175 \times \frac{125}{6} \times 175 \times \frac{50}{9} \times \frac{250}{99} = 15664091727 : 1$. The intermediate harmonic areas between Venus and Mars, and between Mars and Mercury, may, perhaps, be partly distributed among the asteroids, zodiacal meteoroids, the intra-Mercurial harmonic nodes, and the special requirements of approximately synchronous rotation in the dense planetary belt.

If we let $m_0$, $m_3$, $m_5$ represent the masses of Sun, Earth, Jupiter; $v_3$, Earth's aphelion orbital velocity; the value of $m_0$, in units of $m_3$, which would satisfy central requirements of subsidence, linear oscillation, conical oscillation, and the conversion of subsident into orbital velocity, is $(2 \times 3 \times 4)^4 = 331776$. The photodynamic *vis viva* at the chief centre of condensation, $\frac{1}{2}(m_3 v_\lambda^2)$, furnishes the following approx. harmonic proportionality:–

$$m_0 v_3 v_\lambda : m_3 v_\lambda^2 :: m_3 v_\lambda^2 : m_5 v_3 v_\lambda. \qquad (2)$$

For, $\zeta_3 = (331776)^{1/3} \times 3962.8 \times (31558149 \div 5073.8)^{2/3} = 92783200$ miles; $v_\lambda = \zeta_3 \div 497.827 = 186376$ miles; $v_3 = 2\pi\zeta_3 \div (1\ 01677 \times 31558149) = 18.1683$ miles. Substituting these values in (2) we get, $m_5 = 317.18$; $m_0 = 1046.02$. A harmonic approximation which is, probably, still closer, is found by taking $r_0$ as the locus of the secular perihelion centre of gravity of the two controlling masses, $m_0$ and $m_5$; Stockwell's estimate (*loc. cit.*), with the British Nautical Almanac estimate of Sun's semidiameter, gives $.9391726 \times 5.202798 \times 214.45 = 1047.872$.

The maximum and mean planetary accelerations and retardations at secular and mean apsides, produce æthereal disturbances with secondary nodal tendencies, which modify the harmonic areas in various ways. The masses of the principal planets in the extra-asteroidal and intra-asteroidal belts, Jupiter and Earth, are determined, as we have seen, by simple harmonic relations to Sun. Their companion planets, Saturn and Venus, show the nodal influence of the principals; Jupiter and Saturn varying directly as the mean orbital *vis viva*, and inversely as the mean subsidence potential, of particles in their respective orbits, while Earth and Venus represent the maximum disturbance of the mean orbital *vis viva* of Earth's particles upon the orbital *vis viva* of the particles of Venus. Uranus shows the combined influence of apsidal nodes of Earth, Jupiter and Uranus; Neptune, the combined influence of apsidal nodes of Earth Uranus and Neptune.

Let $\alpha$, $\beta$, $\gamma$, $\delta$, $\varepsilon$ represent, respectively, secular aphelion, mean aphelion, mean, mean perihelion, secular perihelion; subscript 0, 2, 3, 5, 6, 7, 8, respectively, Sun, Venus, Earth, Jupiter, Saturn, Uranus, Neptune. Then the harmonic mass-ratios will be represented by the following proport'ns:—

$$m_3 : m_2 :: \gamma_3 : \alpha_2$$
$$m_0 : m_3 :: 24^4 : 1$$
$$m_0 : m_5 :: \varepsilon_5 : \varepsilon_0$$
$$m_5 : m_6 :: \delta_6\gamma_6 : \delta_5\gamma_5$$
$$m_7 : m_3 :: (\beta_7-\varepsilon_3)\gamma_7 : (\beta_7+\delta_5)\gamma_3$$
$$m_8 : m_3 :: (\delta_8-\alpha_3)\gamma_8 : (\delta_8+\alpha_7)\gamma_3.$$

The accordance between the nodal and computed values is shown in the following table:—

| | | Nodal. | Computed. | Authority. |
|---|---|---|---|---|
| Venus | $m_0 \div m_2$ | 428417 | 427240 | Hill. |
| Earth | $m_0 \div m_3$ | 331776 | 331776 | Chase. |
| Jupiter | $m_0 \div m_5$ | 1047.872 | 1047.879 | Bessel. |
| Saturn | $m_0 \div m_6$ | 3503.22 | 3501.6 | Bessel. |
| Uranus | $m_0 \div m_7$ | 22643 | 22600 | Newcomb. |
| Neptune | $m_0 \div m_8$ | 19428 | 19380 | Newcomb. |

The following points of symmetry and alternation may be noted in the nodal mass-factors of the outer planets:—

1. The tendency to equality of mean orbital *vis viva* in Earth, Uranus and Neptune, as indicated by the factors $\gamma_3$, $\gamma_7$, and $\gamma_8$.
2. The nodal modification of Neptune's mass by Earth's secular aphelion, and of the mass of Uranus by Earth's secular perihelion.
3. The nodal modification of Neptune's mass by its own mean aphelion, and of the mass of Uranus by its mean aphelion.
4. The modification of Uranus by Jupiter, and the corresponding modification of Neptune by Uranus.

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## *NOTE ON DIRECTION.*

BY PROFESSOR T. M. BLAKSLEE.

[Ccontinued from page 16.]

WHAT has preceded relative to the point as generatrix and line as path, may assist in giving a clear idea of direction, which is rendered necessary by the increased use of the term in Geometry and Quaternions.